



2005 Access Initiative

Multiple Draws – Statistical Analysis

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Issue

- Can the average production cost savings (due to an access-project) be estimated from a limited number of draws (3-6?) with sufficient (90-95%) confidence?
- Can the average savings be estimated within a 5-10% accuracy of the actual (i.e. true) mean value of the expected savings (from a statistically large population sample of draws)?



‘Point Estimation’ Method

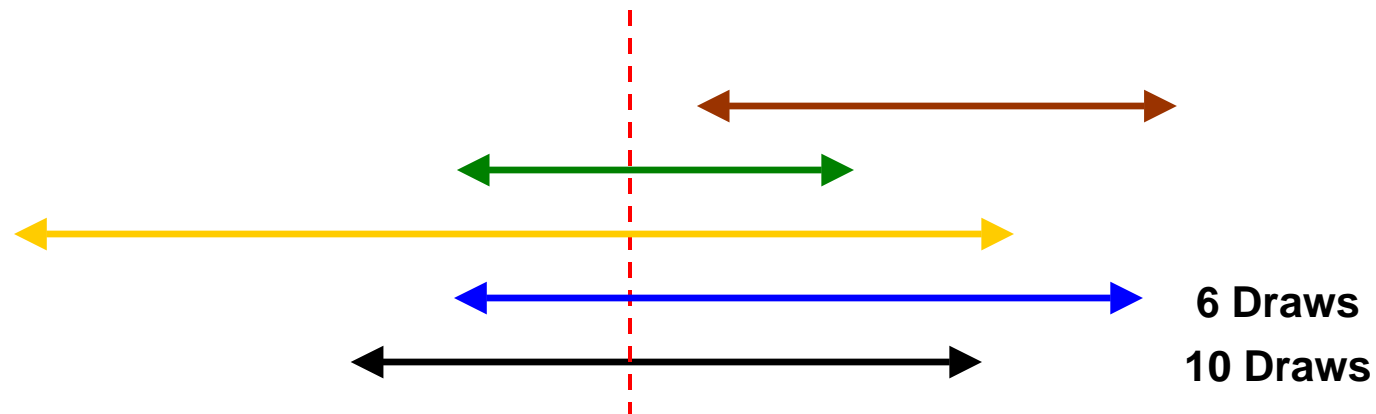
- Calculating Sample Mean (M) and Sample Std-Dev (SD) gives *point estimate* of the True Population Mean (μ) and its True Std-Dev (σ) with known probability distribution (e.g. Normal, Chi-squared, T-, etc.)
- Accuracy in point estimate of sample mean requires large sample size (n) – large n reduces variability of M about μ
- For limited sample size, need several population samples to ensure that the average (expected) value of M estimates (from several samples) lies reasonably close to μ
- Drawback: provides idea of the estimate value, but not of the estimate accuracy!

Is there a better statistical analysis method?



‘Interval Estimation’ Method

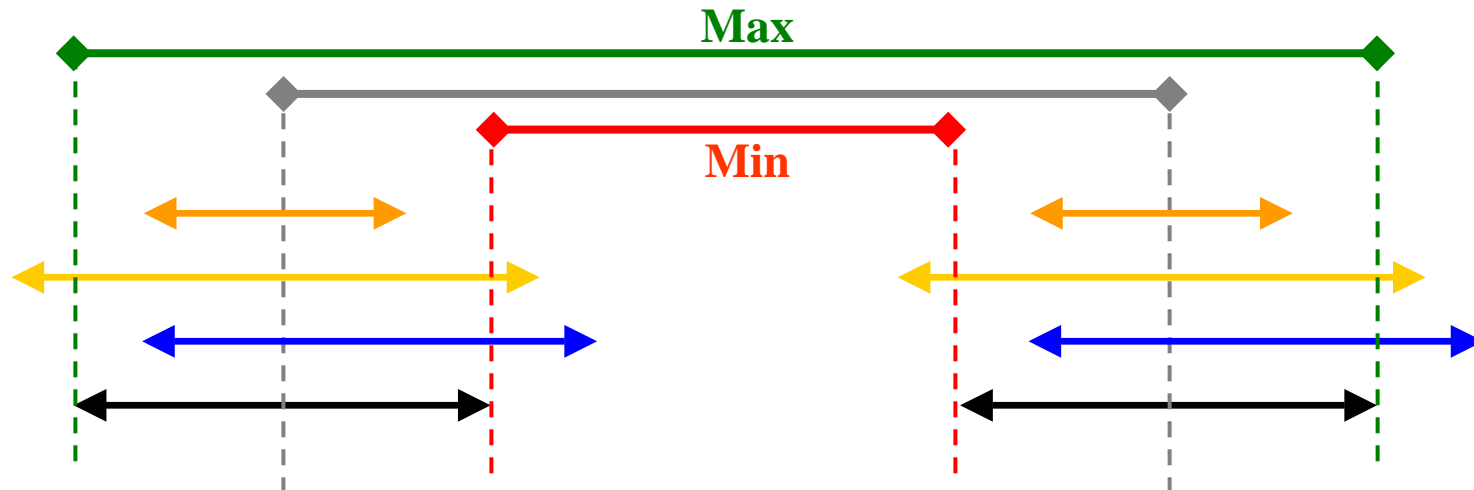
- Estimates the numerical interval for Sample Mean (M) that “captures” the True Mean (μ) of a Population with a certain confidence level (such as 95%) using a limited sample size
 - Do not need several population samples
 - Population probability distribution needs to be approx. known – as sample size n gets larger, the sample mean M distribution is approx. Normal with mean μ and variance σ^2/n
 - 95% confidence interval: in repeated sampling from the population, 95% of the numerical intervals generated are expected to contain μ





Conclusions

- Despite the uncertainty of True Mean location within the 95% confidence interval, the Min/Max limits of achievable Savings can be determined accurately by estimating the PC confidence intervals before and after the access project.
- If the estimated Min Savings are much higher than the project cost, limited sample size (3-6 draws) is not an issue. Only if the two values are comparable (?), additional draws may be justified to increase the sample size and achieve better accuracy in the interval estimates for PC.





Data for Analysis

- Production Costs (PC) from Promod runs for two Y2012 scenarios --- Base-case and with Byron-N.Madison Project
 - For each scenario, 10 random forced outage draws were run to get “large” population sample
 - Smaller population samples of 6 draws used for comparison
 - All production costs (or savings) are in Y2012 dollars
- PC Savings for Draw n =
PC for Base-case Draw n – PC for Project Draw n



Point Estimates

	Production Costs		
Draw	Base Case	Byron-NMA	PC Savings
1	\$1,338,575,544	\$1,315,878,754	\$22,696,790
2	\$1,341,590,525	\$1,319,339,470	\$22,251,055
3	\$1,342,509,318	\$1,318,976,968	\$23,532,351
4	\$1,336,119,824	\$1,313,274,334	\$22,845,491
5	\$1,337,983,479	\$1,311,676,170	\$26,307,309
6	\$1,338,772,721	\$1,319,436,570	\$19,336,151
7	\$1,334,054,278	\$1,317,329,063	\$16,725,215
8	\$1,339,765,942	\$1,316,390,034	\$23,375,908
9	\$1,342,390,378	\$1,322,382,731	\$20,007,647
10	\$1,335,041,227	\$1,313,944,739	\$21,096,488
Sample Mean (M)	\$1,338,680,324	\$1,316,862,883	\$21,817,440
Sample Std-Dev (SD)	\$2,977,287	\$3,289,716	\$2,654,755
Std-Dev,SD / Mean,M	0.2224%	0.2498%	12.1680%
M – 1xSD	\$1,335,703,037	\$1,313,573,167	\$19,162,686
M + 1xSD	\$1,341,657,611	\$1,320,152,599	\$24,472,195
Samples within M+/-1xSD	6	7	8
Samples outside M+/-1xSD	4	3	2

- Variability of PC data is very small compared to that of PC Savings data.
- True Mean of PC data can be estimated with higher confidence, that is, estimated mean within a narrow confidence interval (min/max limits).



Interval Estimation

95% Confidence Intervals for Sample Mean (M)

Population Sample Derived From	95% Confidence Interval for Sample Mean (M) Base Case			95% Confidence Interval for Sample Mean (M) Byr-NM Project			95% Confidence Interval for Sample Mean (M) Savings		
	Lmin	Lmax	Spread	Lmin	Lmax	Spread	Lmin	Lmax	Spread
6 Draws	\$1,337,358,766	\$1,341,158,372	\$3,799,606	\$1,313,732,202	\$1,319,128,554	\$5,396,352	\$21,033,779	\$24,622,603	\$3,588,824
10 Draws	\$1,336,834,982	\$1,340,525,665	\$3,690,683	\$1,314,823,896	\$1,318,901,870	\$4,077,974	\$20,172,006	\$23,462,875	\$3,290,869
6W Draws	\$1,335,421,340	\$1,340,184,536	\$4,763,195	\$1,313,703,672	\$1,319,845,671	\$6,141,999	\$18,421,194	\$23,635,339	\$5,214,145
6B Draws	\$1,336,745,375	\$1,341,996,897	\$5,251,523	\$1,314,475,251	\$1,320,123,748	\$5,648,497	\$21,036,572	\$23,106,702	\$2,070,130

Deriving the Max/Min Limits on True Mean Savings

Population Sample Derived From	Assume True (Population) Means at						Assume True (Population) Means at		
	Lmax for BC & Lmin for Project			Lmin for BC & Lmax for Project			Center of Spread (Lmax+Lmin)/2		
	Base Case	Byr-NM Project	Savings	Base Case	Byr-NM Project	Savings	Base Case	Byr-NM Project	Savings
6 Draws	\$1,341,158,372	\$1,313,732,202	\$27,426,170	\$1,337,358,766	\$1,319,128,554	\$18,230,212	\$1,339,258,569	\$1,316,430,378	\$22,828,191
10 Draws	\$1,340,525,665	\$1,314,823,896	\$25,701,769	\$1,336,834,982	\$1,318,901,870	\$17,933,112	\$1,338,680,324	\$1,316,862,883	\$21,817,441
6W Draws	\$1,340,184,536	\$1,313,703,672	\$26,480,864	\$1,335,421,340	\$1,319,845,671	\$15,575,670	\$1,337,802,938	\$1,316,774,671	\$21,028,267
6B Draws	\$1,341,996,897	\$1,314,475,251	\$27,521,647	\$1,336,745,375	\$1,320,123,748	\$16,621,627	\$1,339,371,136	\$1,317,299,499	\$22,071,637



Observations/Inferences

1. The 95% confidence interval limits for Mean PC Savings varied by approx. 5% with Sample size (10 vs. 6 draws).
Therefore, a 95% confidence interval for Mean PC Savings may be estimated with approx. +/- 5% loss of accuracy by using a sample of 6 draws.
2. The Max/Min limits on the range of achievable PC Savings account for the uncertainty of true mean location within the 95% confidence intervals estimated for the before/after PC.
 - *If the project cost is well below the Min. limit on True Mean, then any estimation error incurred due to small sample size is a moot issue for economic justification of the project.*
 - *Only if the two values are comparable (?), additional draws may be justified to increase the sample size and achieve better accuracy in the interval estimates for PC.*



Conclusions

- Can the average production cost savings (due to an access-project) be estimated from a limited number of draws (3-6?) with sufficient (90-95%) confidence?

Yes for 6 draws; need to evaluate if 3-4 draws may be sufficient.

- Can the average savings be estimated within a 5-10% accuracy of the actual (i.e. true) mean value of the expected savings (from a statistically large population sample of draws)?

Depends if the Sample SD is small enough such that the confidence interval “spread” is <5-10% of Mean.....

determining the min/max of achievable savings is probably a safer metric for economic justification of project!



Data for Analysis

Draw	Production Costs			Cumulative Statistics of PC Savings		
	Base Case	Byron-NMA	PC Savings	Mean, Mn	Std-Dev, Sn	Sn/Mn
1	\$1,338,575,544	\$1,315,878,754	\$22,696,790	\$22,696,790	Undefined	NA
2	\$1,341,590,525	\$1,319,339,470	\$22,251,055	\$22,473,922	\$315,182	1.40%
3	\$1,342,509,318	\$1,318,976,968	\$23,532,351	\$22,826,732	\$650,456	2.85%
4	\$1,336,119,824	\$1,313,274,334	\$22,845,491	\$22,831,422	\$531,178	2.33%
5	\$1,337,983,479	\$1,311,676,170	\$26,307,309	\$23,526,599	\$1,621,102	6.89%
6	\$1,338,772,721	\$1,319,436,570	\$19,336,151	\$22,828,191	\$2,242,547	9.82%
7	\$1,334,054,278	\$1,317,329,063	\$16,725,215	\$21,956,337	\$3,084,113	14.05%
8	\$1,339,765,942	\$1,316,390,034	\$23,375,908	\$22,133,784	\$2,899,108	13.10%
9	\$1,342,390,378	\$1,322,382,731	\$20,007,647	\$21,897,546	\$2,802,944	12.80%
10	\$1,335,041,227	\$1,313,944,739	\$21,096,488	\$21,817,440	\$2,654,755	12.17%
Sample Mean (M)	\$1,338,680,324	\$1,316,862,883	\$21,817,440			
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Samples within M+/-1xSD	6	7	8			
Samples outside M+/-1xSD	4	3	2			



Analysis Results

Population Derived From	Production Cost -- Sample Mean (M)			Production Cost -- Sample StdDev (SD)		
	Base Case	Byr-NM Project	Savings	Base Case	Byr-NM Project	Savings
6 Draws	\$1,339,258,569	\$1,316,430,378	\$22,828,191	\$2,374,259	\$3,372,018	\$2,242,547
10 Draws	\$1,338,680,324	\$1,316,862,883	\$21,817,440	\$2,977,287	\$3,289,716	\$2,654,755
6W Draws	\$1,337,802,938	\$1,316,774,671	\$21,028,267	\$2,976,377	\$3,837,950	\$3,258,162
6B Draws	\$1,339,371,136	\$1,317,299,499	\$22,071,637	\$3,281,518	\$3,529,576	\$1,293,562

- When sample size is reduced from 10 to 6 draws
 - Sample Mean estimate for PC Savings changes from \$21.817M to \$22.828M
 - Sample Std-Dev estimate for PC Savings changes from \$2.655M to \$2.242M
- 6W draws sample was chosen to get worse Std-Dev of PC Savings – sample mean deviates farther away from M for 10 draws
- 6B draws sample was chosen to get best Std-Dev of PC Savings – sample mean deviates minimally from M for 10 draws